OCR RECOGNISING ACHIEVEMENT

Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A camshaft inside an engine is rotating with angular speed $42\,\text{rad}\,\text{s}^{-1}$. When the throttle is opened the camshaft speeds up with constant angular acceleration, and 8 seconds after the throttle was opened the angular speed is $76\,\text{rad}\,\text{s}^{-1}$.

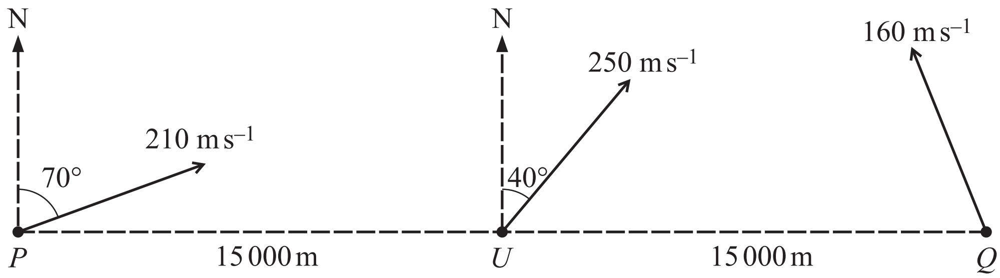
(i) Find the angular acceleration of the camshaft. **[2]**

(ii) Find the time taken for the camshaft to turn through 810 radians from the moment that the throttle was opened. **[3]**

2 A straight rod AB has length a. The rod has variable density, and at a distance x from A its mass per unit length is given by $k\left(4-\sqrt{\frac{x}{a}}\right)$, where k is a constant. Find the distance from A of the centre of mass of the rod. **[7]**

3 The region R is bounded by the x-axis, the y-axis, the curve $y = ae^{\frac{x}{a}}$ and the line $x = a\ln 2$ (where a is a positive constant). A uniform solid of revolution, of mass M, is formed by rotating R through 2π radians about the x-axis. Find, in terms of M and a, the moment of inertia about the x-axis of this solid of revolution. **[8]**

4



An unidentified aircraft U is flying horizontally with constant velocity $250\,\text{m}\,\text{s}^{-1}$ in the direction with bearing 040°. Two spotter planes P and Q are flying horizontally at the same height as U, and at one instant P is 15 000 m due west of U, and Q is 15 000 m due east of U (see diagram).

(i) Plane P is flying with constant velocity $210\,\text{m}\,\text{s}^{-1}$ in the direction with bearing 070°.

(a) Find the magnitude and bearing of the velocity of U relative to P. **[4]**

(b) Find the shortest distance between P and U in the subsequent motion. **[2]**

(ii) Plane Q is flying with constant velocity $160\,\text{m}\,\text{s}^{-1}$ in the direction which brings it as close as possible to U.

(a) Find the bearing of the direction in which Q is flying. **[4]**

(b) Find the shortest distance between Q and U in the subsequent motion. **[2]**

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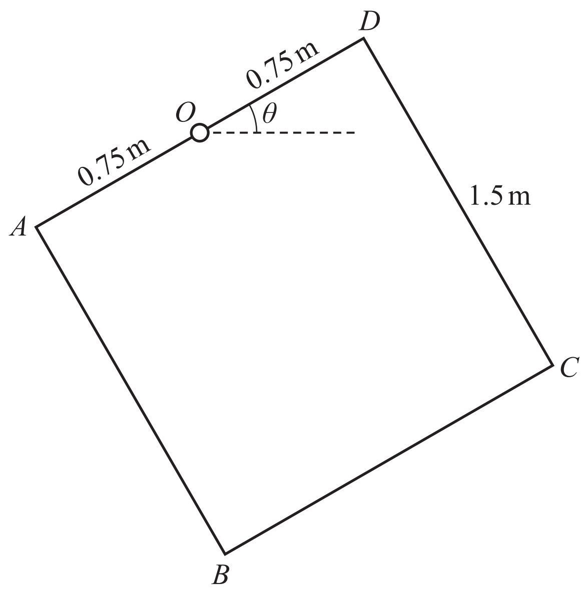


A square frame *ABCD* consists of four uniform rods *AB*, *BC*, *CD*, *DA*, rigidly joined at *A*, *B*, *C*, *D*. Each rod has mass 0.6 kg and length 1.5 m. The frame rotates freely in a vertical plane about a fixed horizontal axis passing through the mid-point *O* of *AD*. At time *t* seconds the angle between *AD* and the horizontal, measured anticlockwise, is θ radians (see diagram).

(i) Show that the moment of inertia of the frame about the axis through *O* is $3.15\,\mathrm{kg\,m^2}$. **[4]**

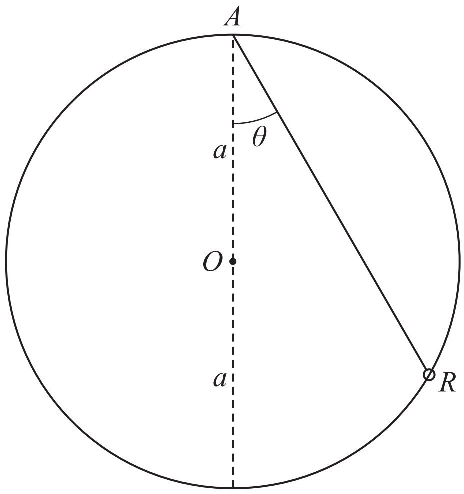
(ii) Show that $\dfrac{d^2\theta}{dt^2} = -5.6\sin\theta$. **[3]**

(iii) Deduce that the frame can make small oscillations which are approximately simple harmonic, and find the period of these oscillations. **[3]**

The frame is at rest with *AD* horizontal. A couple of constant moment 25 N m about the axis is then applied to the frame.

(iv) Find the angular speed of the frame when it has rotated through 1.2 radians. **[4]**

6



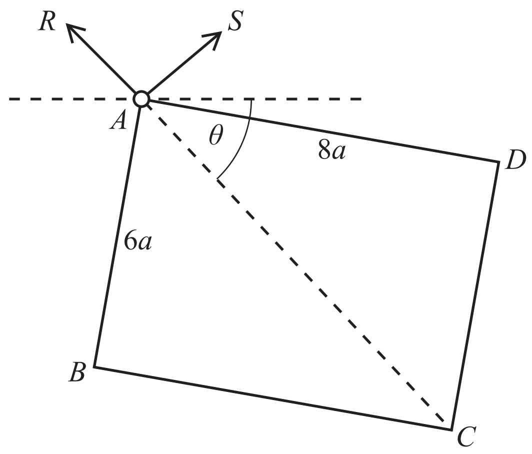
A smooth wire forms a circle with centre O and radius a, and is fixed in a vertical plane. The highest point on the wire is A. A small ring R of mass m moves along the wire. A light elastic string, with natural length $\frac{1}{2}a$ and modulus of elasticity $2mg$, has one end attached to A and the other end attached to R. The string AR makes an angle θ (measured anticlockwise) with the downward vertical (see diagram), and you may assume that the string does not become slack.

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy of the system is $mga(6\cos^2\theta - 4\cos\theta + \frac{1}{2})$. **[4]**

(ii) Show that there are two positions of equilibrium for which $0 \leqslant \theta < \frac{1}{2}\pi$. **[4]**

(iii) For each of these positions of equilibrium, determine whether it is stable or unstable. **[4]**

7



$ABCD$ is a uniform rectangular lamina with mass m and sides $AB = 6a$ and $AD = 8a$. The lamina rotates freely in a vertical plane about a fixed horizontal axis passing through A, and it is released from rest in the position with D vertically above A. When the diagonal AC makes an angle θ below the horizontal, the force acting on the lamina at A has components R parallel to CA and S perpendicular to CA (see diagram).

(i) Find the moment of inertia of the lamina about the axis through A, in terms of m and a. **[3]**

(ii) Show that the angular speed of the lamina is $\sqrt{\dfrac{3g(4+5\sin\theta)}{50a}}$. **[3]**

(iii) Find the angular acceleration of the lamina, in terms of a, g and θ. **[2]**

(iv) Find R and S, in terms of m, g and θ. **[6]**

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